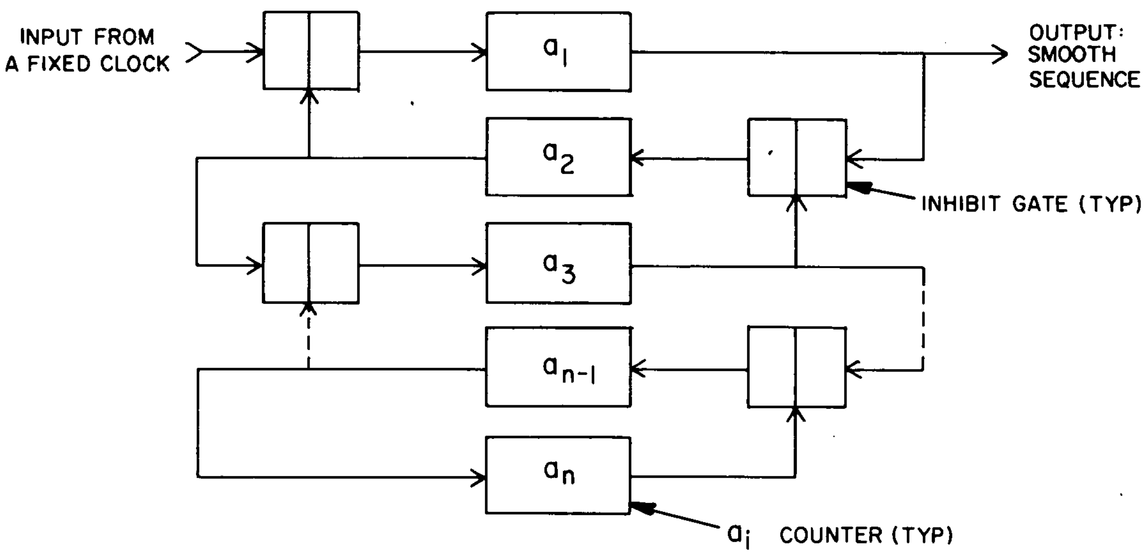
NASA TECH BRIEF

Manned Spacecraft Center

A Continued Fraction Generator for Smooth Pulse Sequences



Continued Fraction Generator

The problem:

To develop a simplified digital circuit to produce a rational output pulse rate at a fraction of a continuous input pulse rate. In addition, the output pulses must be smooth, i.e., have an average rate with the least possible deviation from the absolute correct time spacing.

The solution:

A digital circuit consisting of counters which produce one pulse for every a_i of input pulses received, and inhibit gates which set as conductors unless a pulse appears on the inhibit line. In which case, the next input pulse is blocked but reset for the following pulse. The value of the a_i's are determined from an algorithm.

How it's done:

Let A/B be a given pulse rate when A is the number of output pulses desired and B is the number of input pulses. A/B is given in lowest terms and $0 < A/B < 1$. The a_i's are simply the coefficients of the simple continued fraction for A/B and can be derived from the Euclidean Algorithm. To find the a_i's, let the quotient be a a_i and the remainder r_1.

$$\frac{B}{A} = a_1 + r_1 \text{ where } 0 \leq r_1 < A$$

Next, divide A by r_1,

$$\frac{A}{r_1} = a_2 + r_2 \text{ where } 0 \leq r_2 < r_1$$

(continued overleaf)

And

$$\frac{r_1}{r_2} = a_3 + r_3 \text{ where } 0 \leq r_3 < r_2$$

Then in general,

$$\frac{r_{n-3}}{r_{n-2}} = a_{n-1} + r_{n-1}$$

And,

$$\frac{r_{n-1}}{r_{n-1}} = a_n \text{ (} r_n \text{ is the first remainder to be zero)}$$

Finally, the rate A/B can be described by:

$$\frac{A}{B} = \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cdots \cfrac{1}{a_{n-1} + \cfrac{1}{a_n}}}}}}$$

Notes:

1. This configuration is believed to be, on the average, more economical than previous devices, yet has all the advantages of the previous configurations.
2. Smooth pulse sequences are described by A. J. Lincoln, S. Even, and M. Cohn in "Smooth Pulse Sequences, "Proceeding of the Third Annual Princeton Conference on Information Sciences Systems, 1969.
3. Possible uses for this circuit include frequency synthesizing, the generation of fractions, and the approximation of irrational sequences.
4. Requests for further information may be directed to:

   Technology Utilization Officer
   Manned Spacecraft Center, Code JM7
   Houston, Texas 77058
   Reference: TSP71-10304

Patent status:

No patent action is contemplated by NASA.

Source: Martin Cohn, S. James Lincoln and
Shimon Even
Sperry Rand
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